Geometry Optimization of a 2.45 GHz Microwave Electrothermal Thruster Resonant Cavity

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SP2014_2980822

Abstract

Proper electric field distribution in the resonant cavity is very important for initiating and maintaining the plasma discharge in a Microwave Electrothermal Thruster system. If electric field is not strong enough, electrons will not absorb adequate energy and the plasma will quench. Electric field reaches its peak value in the case of the incident wave frequency being equal to the resonant frequency of the cavity. In order to reach maximum field strength in the medium inside the cavity the disturbance factors for microwave emission must be compensated. The dielectric plate that separates the microwave applicator domain from plasma domain provides such a disturbance. Additionally, the plasma itself acts as a reflecting surface after reaching cutoff level electron density. Since the applied frequency is constant (at 2.45 GHz), thus the resonant frequency has to be altered by changing the dimensions of the resonant cavity. In this study cavity dimension optimization of MET resonant cavity is done by using COMSOL Multiphysics commercial software. Simulations are made for cavity fed with coaxial port which has power input of 1 kW and 2.45 GHz frequency. Cavity radius is maintained constant 50.8 mm and height changing in 80 mm range with 1 mm steps for parametric sweep. The effects of non-uniformities mentioned above are studied and the results are presented.

I. Introduction

Microwave Electrothermal Thruster (MET) is an in space propulsion system that employs electromagnetic energy to heat up propellant gas. MET consists of three main parts; microwave applicator, resonant cavity and the nozzle. Conversion of microwave energy into thermal energy of the gas is materialized in three steps;

1. Free electrons in the propellant gas are accelerated via the electric field component of microwave in accordance with the Lorentz Force (**F**=q**E**) where q is the elementary charge and **E** is the electric field.

2. Energized electrons transfer their energy to heavy particles (ions, molecules and atoms) after each collision. If the transferred energy is adequate to ionize neutrals, number of electrons will increase. When this number reaches the critical point plasma discharge will start.

3. Propellant gas heat up when swirling around free floating plasma as shown in Figure 1 When gas flows through de Laval nozzle it expands and thermal energy of the gas is transferred into kinetic energy to obtain thrust [1].

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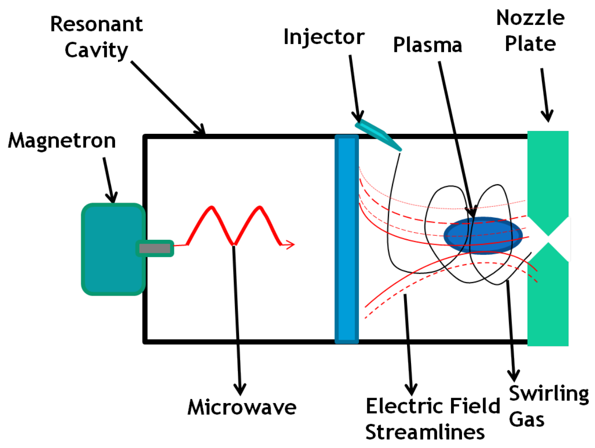


Figure 1: Microwave Electrothermal Thruster System

Microwave thruster system was conceived in order to overcome the handicaps of Resistojets and DC Arcjets. Namely, the thermal endurance of the heater is the performance limiting factor for resistojets, MET handles this problem by using free floating plasma instead of solid heater. The electrode corrosion is the lifetime limiting problem for Arcjets. On the other hand in a MET, plasma arc is generated by transmitting microwave power directly into the flow region via antenna out of this region. MET prototypes, produced to date, demonstrate very close performance characteristics with Arcjets and Resistojets as shown in Table 1.

Table 1: Comparison of Electrothermal Thrusters [2]

System	Power [kW]	Thrust [mN]	Vacuum Isp [s]	Efficiency
Resistojet	0.5-1.5	5-500	300-350	0.80
Arcjet	0.005-26	50-5000	150-1000	0.35
MET	0.07-5	2-700	150-600	0.50

Employing microwave resonant cavity as a part of thruster system was conducted by Michigan State University researchers firstly in 1982 although it was an idea of 1960s. Efficiency of the system is one of the main issues which caused this delay. Since the first prototype built in early 90s, scientists engaged in determining limit of system coupling efficiency to built up efficient thruster [3]. System efficiency limit is related to the amount of microwave power absorbed by propellant gas [4]. Neglecting loses to the cavity walls coupling efficiency can be expressed:

$$\frac{P_i - P_r}{P_r} \times 100 \tag{1}$$

where P_i is the incident power and P_r is the reflected power. Proper electric field strength is crucial for initiating and maintaining the plasma discharge. Electric field reaches its peak value when the reflections are minimum. At the resonant conditions, reflections diminish significantly. In order to reach maximum field strength in the medium inside the cavity the disturbance factors for microwave absorption must be compensated. The dielectric plate that separates the microwave applicator domain from plasma domain provides such a disturbance. Additionally, the plasma itself acts as a reflecting surface after reaching cutoff level electron density.

In this study, cavity dimension optimization of MET resonant cavity is done by COMSOL Multiphysics commercial software. Simulations are made for cavity fed with coaxial port which has power input of 1 kW and 2.45 GHz frequency. Cavity radius is maintained constant 50.8 mm and height of the cavity is varied for tuning. The effects of non-uniformities mentioned above are studied and the results are presented.

II. Theory

In free space electromagnetic waves propagate in TEM (Transverse Electromagnetic) Mode. Both electric and the magnetic field components are orthogonal to the direction of propagation and each other. But in real life electromagnetic waves are bounded between media with different characteristics and propagate in TE (Transverse Electric mode: no electric component in the direction of propagation) or TM (Transverse Magnetic mode: no magnetic component in the direction of propagation) [5].

Designing a MET, electric field intensity is desired to be at its highest value on the axis of the cavity and near the nozzle to reduce thermal loses to the walls. Experiments to date have shown that TM_{011} mode best fits the microwave electro thermal concept [6, 7]. For TM_{011} mode electric field components in cylindrical cavity can be written as;

$$E_z = A_{01} J_0 \left[\frac{\chi_{01}}{a}\rho\right] cos \left[\frac{\pi}{h}\right] \tag{2}$$

$$E_\rho = A_{01} \frac{\pi a}{\chi_{01} h} J_1 \left[\frac{\chi_{01}}{a}\rho\right] sin \left[\frac{\pi}{h}\right] \tag{3}$$

$$E_\phi = 0 \tag{4}$$

Where J_0 is the Bessel function of the first type of *zeroth* order. χ_{01} is the first zero of this Bessel function. Relation between cavity dimensions and the resonant frequency can be expressed as below;

$$(f_r)_{TM_{011}} = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{(\frac{\chi_{01}}{a})^2 + (\frac{\pi}{h})^2} \tag{5}$$

Height of the cavity can be determined by solving the resonant frequency equation for a specific radius. Another important parameter is the quality factor which gives the ratio between the average energy stored W in the cavity to energy dissipated in the cavity δW [5, 8];

$$Q = \frac{W}{\delta W} \tag{6}$$

Namely, Q is very high and the most of the incident power is reflected before plasma ignition and reduces to $Q \backsimeq 1$ after ignition due to the absorption by the plasma. Nearly all of the wave power dissipated by the collisions between electrons and heavy particles [3]. As mentioned above empty resonant cavity has a very high quality factor, after the discharge started it acts as a resistive load inside the cavity and dissipate the energy stored in the cavity. Power absorbed by the plasma can be expressed as;

$$P_{ave} = \frac{n_e e^2 E^2}{m} \frac{\nu_m}{\nu_m^2 + \omega^2} \tag{7}$$

Where E is the electric field strength[6][9]. The electromagnetic field equations can be derived from Maxwells' equations as below;

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \tag{8}$$

$$\nabla \times \mathbf{H} = j\omega\epsilon_0\epsilon_r\mathbf{E} + \mathbf{J} \tag{9}$$

$$\mathbf{J} = \sigma\mathbf{E} \tag{10}$$

$$\nabla \times \frac{1}{\mu_r}\nabla \times \mathbf{E} - k_0^2(\epsilon_r - \frac{j\sigma}{\epsilon_0\omega})\mathbf{E} = 0 \tag{11}$$

Permittivity and conductivity of the plasma can be introduced as;

$$\epsilon_{rp} = 1 - \frac{\omega_p^2}{\omega(\omega - j\nu_m)} \tag{12}$$

$$\sigma = \frac{e^2 n_e}{m_e} \frac{1}{\nu_m + j\omega} \tag{13}$$

The higher the electron density the higher the conductivity of the plasma[10]. Plasma frequency is proportional to electron number density by $\omega_p = (\frac{e^2 n_e}{\varepsilon_0 m_e})^{1/2}$. Collision frequency for momentum loss can be evaluated by using the equations and the empirical identities [11] below;

$$\nu_m = n_i \left\langle \sigma_{ei} v_e^{th} \right\rangle + n_a \left\langle \sigma_{ea} v_e^{th} \right\rangle \tag{14}$$

In this study we assume that the plasma is quasi neutral, $n_i = n_e$ and the local thermodynamic equilibrium is reached. Total number density $N = 2n_i + n_a$ can be introduced by using ideal gas relation;

$$N = \frac{p}{kT} \tag{15}$$

In experiments to date electronic temperatures are determined at the level of 1 eV [12, 13, 14]. $\left\langle \sigma_{ei} v_e^{th} \right\rangle, \left\langle \sigma_{ea} v_e^{th} \right\rangle$ represents rate constants and evaluated by using the relations given by Jonkers et al [15].

III. Numerical Simulation and Results

As mentioned above, proper cavity design is very important in order to maintain discharge and decrease reflections. We used D. E. Clemens' cavity model for calculations as presented in Figure 2. The model consist of a cylindrical cavity with perfect conductor walls, a feeding port, quartz plate in the middle of the cavity and plasma domain dimensions of which are tabulated in Table 2 [2]. Plasma dimensions are taken from experiments conducted to date [16]. In simulations teflon part of the port had relative permittivity of 2, quartz plate had 4.2, plasma zone except plasma domain had 1 and plasma domain permittivity introduced with the relations in equation 12. COMSOL multi physics RF module is used. Mesh with triangular elements is chosen and mesh consists of 380000 degrees of freedom.

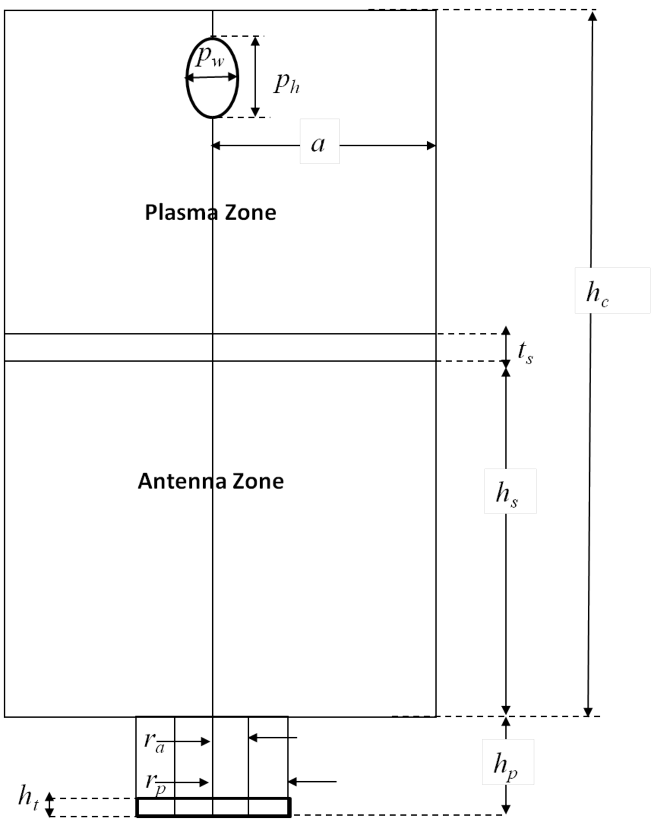


Figure 2: Clemens' Cavity Model

In order to understand the disturbance effect of quartz plate and plasma, and also to determine the optimum cavity height under these conditions, simulations for various h_s values changing 10 to 80 mm with 1 mm intervals have been done. To understand the plasma effect more clearly, simulations for different electron number densities changing 10^{16} to $10^{19} m^{-3}$ are conducted. Tuning effect is determined by analyzing the S-parameter which gives the ratio of reflected power to incident power.

Table 2: Cavity Dimensions

Parameter	Length [mm]
a	50.8
h_c	157.5
h_p	21.89
h_t	5.08
t_s	6.35
r_p	20.9
r_a	8.45
h_s	75.58
p_w	16
p_h	30

In the first analysis we run simulator for empty cavity. At 2.45 GHz, electric field distribution is well matched with the analytic solution for electric field distribution in TM_{011} mode as in Figure 3.

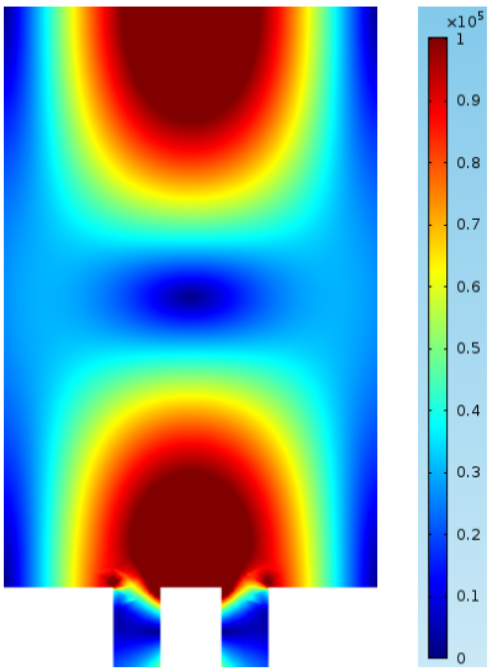


Figure 3: Electric field [V/m] distribution in empty cavity at 2.45 [GHz]

Separation plate is deployed between antenna and plasma zone in the second simulation. Employing quartz plate affected electric field distribution as in Figure 4 and reduced electric field intensity significantly at both ends of the cavity . Another simulation is conducted to determine the tuning effect. As depicted in Figure 5 S-parameter decreased its minimum value when h_s is between 62-63 mm at 2.45 GHz. The system will catch resonance conditions when the wall of the cavity is fixed at this point. When we tune the system electric field intensity increases significantly nearly to the value for empty cavity as seen in Figure 6. This results point out the importance of tuning and optimization of the cavity height for discharge ignition. Namely, in case of low electric field intensity, plasma discharge can not be started or maintained depending on the gas pressure in the cavity according to the Paschen curve which introduces the relation between electric field and pressure of the gas [17][18]. If the gas pressure is high, electrons need to gain enough energy in short mean free path to ionize the gas before two successive collisions. In case of low electric field density energy level of electrons cannot be reached at the point of gas ionization. Plasma will quench consequently.

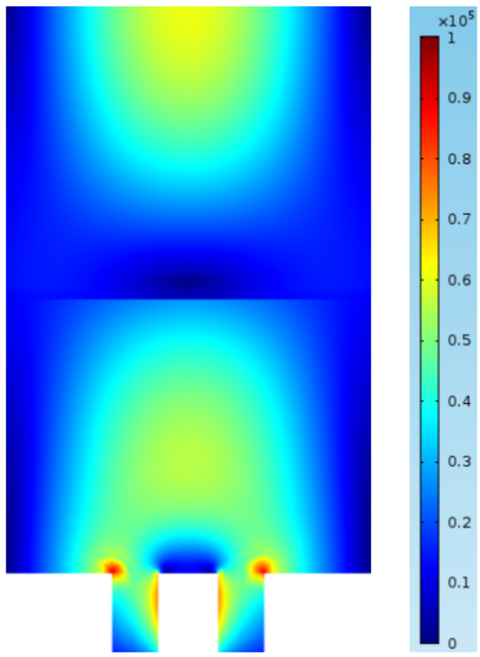


Figure 4: Electric field [V/m] distribution with quartz plate for $h_s = 75.58$ mm

Plasma will absorbs power until it reaches the critical point which can be defined as critical plasma frequency $f_c \backsimeq 9000\sqrt{n_{ec}}$ [3]. Where n_{ec} is the electron number density per cc. Beyond this point it will start to reflect microwave power and the reflections will increase with increasing plasma density. It is very hard to determine the effect of such disturbance analytically. After the plate effect is computed, disturbance due to plasma is tried to be established in the third case. Firstly the effect of number density is tried to be analyzed at constant frequency of 2.45 GHz and the h_s is set to 75.58

mm.

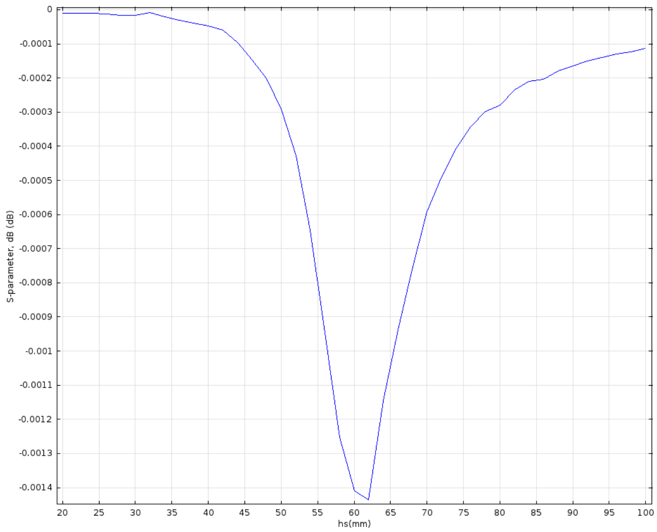


Figure 5: S-parameter vs. h_s for cavity with separation plate

Figure 7 represents electric plasma distribution for the following values of n_e=$10^{16}, 5 \times 10^{16}, 10^{17}, 5 \times 10^{17}, 10^{18}, 5 \times 10^{18}, 10^{19}, 5 \times 10^{19}$ m^{-3}. Plasma effect on the distribution increases with increasing electron number density as seen in Figure 7. Below $n_e = 5 \times 10^{17}$ m^{-3} as seen in Figure 7 (a)(b)(c) plasma is transparent to electromagnetic wave. Beyond this level plasma adversely affect reflection as Figure 7 (d)(e)(f)(g) and electric field distribution in the cavity is changed significantly due to the skin effect of the plasma. To evaluate optimum h_s values for different electron number densities parametric sweep for different values for all number densities. Minimum reflection is reached at n_e=$5 \times 10^{16} m^{-3}$ and h_s=68 mm as in Figure 8.

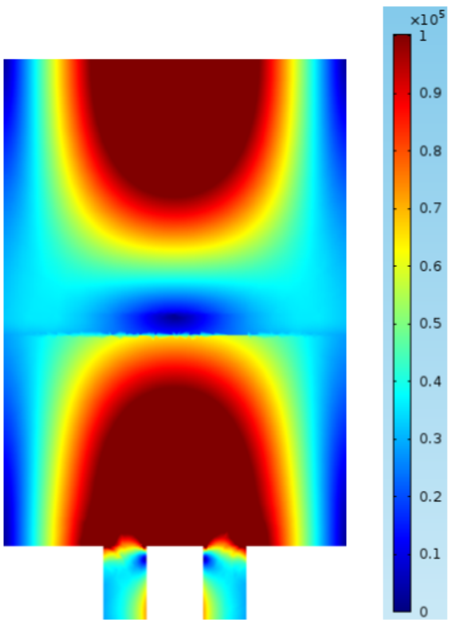
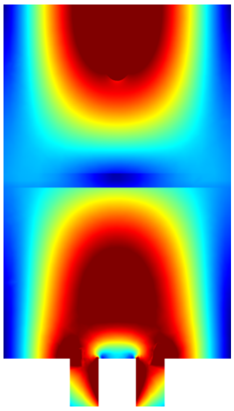
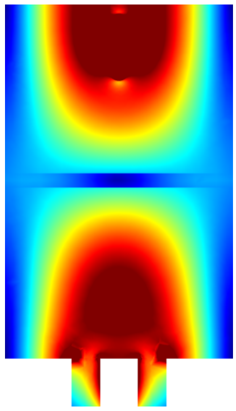


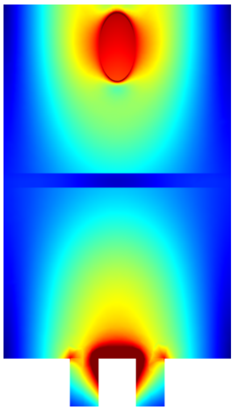
Figure 6: Electric field [V/m] distribution for the cavity with quartz plate for h_s= 62 mm



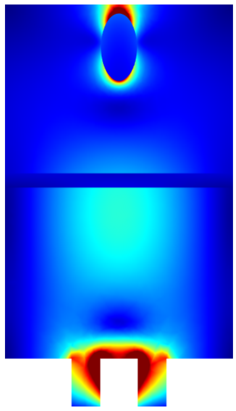
(a) $n_e = 10^{16}$



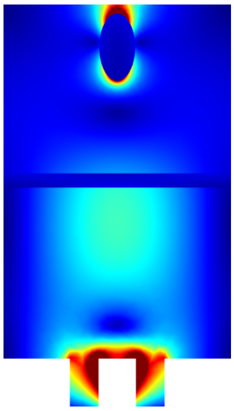
(b) $n_e = 5 \times 10^{16}$



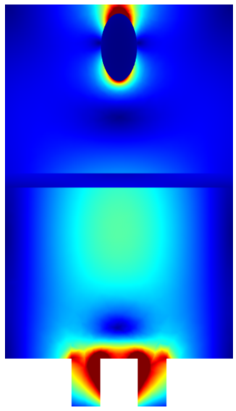
(c) $n_e = 10^{17}$



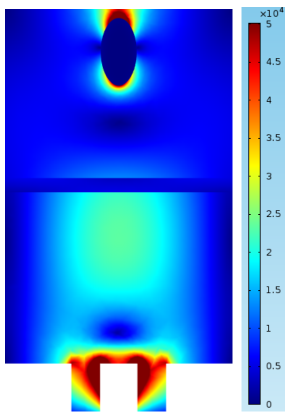
(d) $n_e = 5 \times 10^{17}$



(e) $n_e = 10^{18}$



(f) $n_e = 5 \times 10^{18}$



(g) $n_e = 10^{19}$

Figure 7: Electric field [V/m] distribution for cavity with plasma disturbance at 2.45 GHz for various n_e level

To see the effect of number density on the cavity change of minimum h_s with number density is given in Figure 9 and also change of S-parameter according to number density is given in Figure 10. This simulations show that to produce more efficient resonant cavity with minimum reflections we must adjust the cavity dimensions properly for changing plasma density.

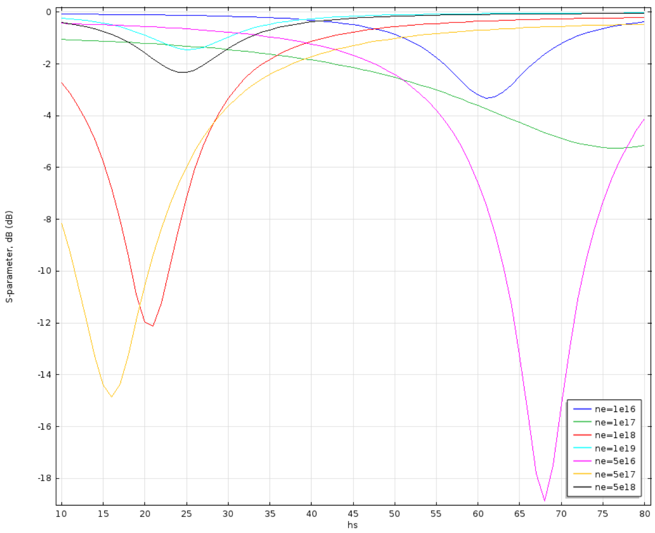


Figure 8: S-parameter vs. h_s for different number densities

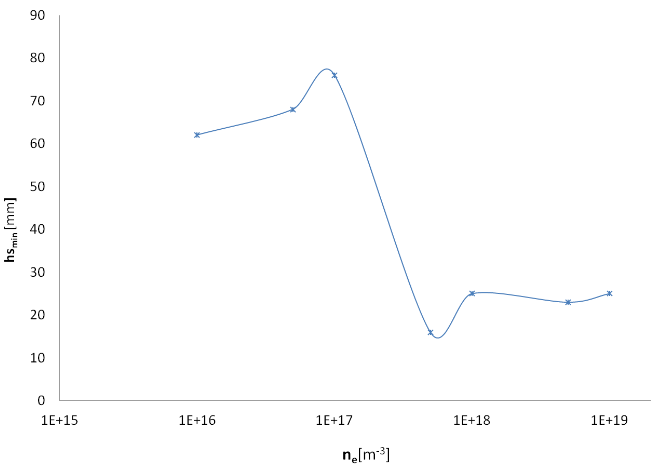


Figure 9: Minimum h_s values for different number densities

IV. Conclusion

Disturbance effect of dielectric plate and the plasma discharge on the wave propagation inside Microwave Electrothermal Thruster cavity is studied. According to the simulation results, it is determined that the plasma effect on the electric field distribution is remarkable. If electron density is high enough plasma can be acted as a lossy conductor. For electron densities greater than $5 \times 10^{18} m^{-3}$ electromagnetic waves partially cannot penetrate into the plasma because of the skin effect. Adjusting the system to optimum h_s value will augment electric field intensity in the cavity and increase potential of plasma discharge ignition.

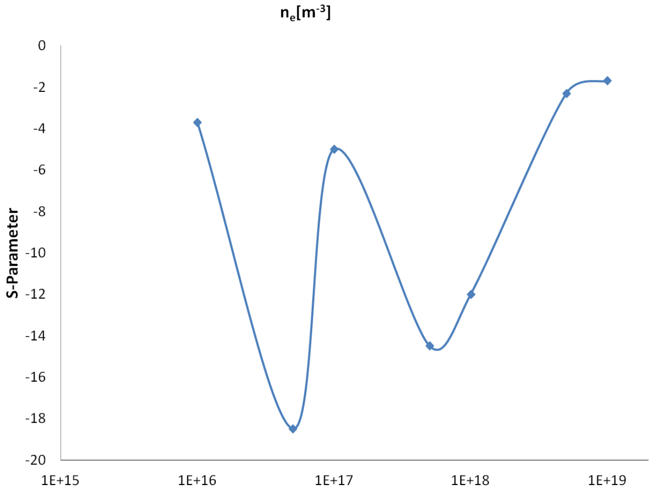


Figure 10: Minimum S-parameter values for different number densities

Acknowledgment

Authors would like to thank Emre Turkoz for his valuable contribution to this study. This research is supported by Turkish Scientific and Technological Reseach Council (TUBITAK) under projects 112M862 and 113M244 and partially by Bogazici University Scientific Projects Office under project number BAP-6184.

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